Title:
Thin sheets and sandpiles: subtle self-organization in prosaic places.

Overall abstract:
These three lectures deal with two forms of conventional macroscopic matter where striking forms of self-organization have been recently discovered. They are of interest because they qualitatively extend our notion of what matter in general can do. The first two lectures deal with thin elastic sheets and their singular response to forcing when the thickness becomes arbitrarily small. The third lecture considers the distinctive nature of solidity in assemblies of impenetrable particles concentrated to the point where they transmit mechanical forces.

Lecture I: Singular structure in thin sheets I

These three lectures deal with two forms of conventional macroscopic matter where striking forms of self-organization have been recently discovered. They are of interest because they qualitatively extend our notion of what matter in general can do. The first two lectures deal with thin elastic sheets and their singular response to forcing when the thickness becomes arbitrarily small. The most obvious singular response is immediately evident when we crumple a piece of office paper between our hands. Though the forcing is uniform and structureless, the sheet deformation is concentrated into sharp points and connecting ridges: elastic energy is condensed into an arbitrarily small fraction of the sheet. The first lecture surveys these condensation phenomena with an eye to characterizing the singular structures at the most basic level. We explain how much the energy is concentrated and why. Using scaling arguments we account for the condensation of energy into one-dimensional ridge structures. As the thickness diminishes, the degree of condensation increases as the inverse 1/3 power of this thickness. We then consider a variety of forced sheets and their resulting singular structure, noting the degree of energy condensation in each. The structures considered are the wrinkle to fold transition, confinement in a tube, induced ridges at the boundary of a sheet, and the developable cone structure. We conclude by noting some unexplained regularities and puzzles.

Lecture II: Singular structure in thin sheets II

The second lecture aims to gain deeper insight under this singular structure formation by abstracting and generalizing the sheet. We begin by noting the ingredients that give rise to these singularities: a manifold with an intrinsic distance scale is embedded in a higher dimensional space and then constrained or forced. These ingredients can apply to general spatial dimensionalities, but the effect of forcing is different depending on the case. Condensation of energy into ridges happens only when the embedding space has one more dimension than that of the deformed manifold. As the dimension of the embedding space increases, the condensation steadily weakens. Condensation ceases when the embedding space has at least double the dimensionality of the manifold. Returning to real two-dimensional sheets, we discuss recent work aimed at understanding two puzzles: the size of the stretched core region in a developable cone and the vanishing of mean curvature at the rim of such a cone. We then discuss the entirely different deformations that result from internal deformations of the distance scale or metric, first identified by Sharon et al [1]. We conclude by speculating how the distinctive concentrations of energy in distorted thin sheets can be compared to the standard types of singularity seen in physical systems. We ask how the thin-sheet phenomena might extend to further forms of matter in which local distances are constrained.

[1] "Shaping of Elastic Sheets by Prescription of Non-Euclidean Metrics" Yael Klein, Efi Efrati, Eran Sharon* Science, vol 315 p. 116 (2007)

Lecture III: Forces in marginally-connected solids.

The third lecture considers the distinctive nature of solidity in assemblies of impenetrable particles concentrated to the point where they transmit mechanical forces. Such assemblies are called granular packs. The solidity of a granular pack is qualitatively different from an elastic solid. We trace this difference to the marginal connectivity of the network of force-bearing contacts. We contrast this marginal connectivity with that seen in a conventional percolation system. We explain this reason for marginal connectivity in the context of a simple sequential packing. We then show how marginal connectivity leads to a different constitutive law than that of an elastic solid. Accordingly the network's response to an applied point force is qualitatively different. Instead of spreading uniformly from the source, the response forces are concentrated in rays. Marginality also creates instability and sensitivity to disorder. Typically the linear response to an applied local force diverges with distance. We discuss another consequence of marginal connectivity in simulated isotropic packs: a distinctive structure of vibrational normal modes that is qualitatively softer than that of an elastic solid. We address a paradox of explaining the force propagation in an isotropic pack. The marginal nature of the packing should lead to propagation along rays, yet an isotropic pack has no preferred direction that could define the orientation of these rays. We end by describing recent simulations aimed at using granular packs to account for the liquid-to-glass transition.