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OBSERVATION OF KNUDSEN AND GURZHI TRANSPORT REGIMES IN A TWO-DIMENSIONAL WIRE

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Abstract—We have observed electronic Knudsen and Poiseuille flow in a current heating experiment on electrostatically defined wires in (Al,Ga)As heterostructures. Current heating induces an increase in the number of electron–electron collisions in the wire, leading first to an increase (Knudsen regime) and subsequently to a decrease (due to Poiseuille electron flow, and known as the Gurzhi effect) of the resistance of the wire.

In his famous 1909 paper on gas flow through a capillary, Knudsen[1] demonstrated that the pressure drop over the capillary first increases and then decreases with increasing density. The mechanism is that with increasing gas-particle density, the number of interparticle collisions also increases. At low densities (what is now known as the Knudsen transport regime) this leads to increasing dissipation of forward molecular momentum at the capillary walls, while at higher densities laminar Poiseuille flow sets in, which decreases the effective particle–wall interaction.

Because of the analogy between classical diffusive transport of electrons and gas particles, one may anticipate that a similar transition from Knudsen to Poiseuille also occurs in electron transport, where normal electron–electron scattering events (NEES) are the analogue of gas particle collisions. This issue has indeed been pursued since the early 1950's. However, it proved difficult to obtain reliable data[2], because the electron–phonon interaction is much more important than the electron–electron interaction.

So far, only preliminary indications of electronic Knudsen and Poiseuille transport effects have been found[3]. Most experiments were performed on potassium, as an exemplary simple metal. However, the observed changes in the resistance as a function of lattice temperature were limited to about 0.01% of the total resistance, because of the limited impurity mean-free-path l_{imp} and the onset of electron–phonon scattering. Observations of a positive temperature derivative of the resistivity $d\rho/dT$ in potassium wires[4] could be assigned by Movshovitz and Wiser[5] as a Knudsen-like behaviour due to the combination of, relatively infrequent, normal electron–electron and electron–phonon collisions. However, until now there has been no observation of electronic Poiseuille flow. Electronic Poiseuille flow should lead to a negative $d\rho/dT$, a phenomenon predicted by Gurzhi in 1963[6], and generally known as the Gurzhi effect.

Here, we present a study of Knudsen and Gurzhi phenomena in two-dimensional wires, fabricated from high-mobility (Al, Ga)As heterostructures. Using this material to study NEES effects offers several advantages, allowing a clear and unambiguous observation of the Knudsen and Poiseuille flow regimes. The resistance changes caused by NEES processes can be larger than 10% of the total resistance. A full discussion of this work, as well as a detailed theoretical framework for calculations of NEES effects on the resistance for arbitrary electron temperature will be given elsewhere[7].

The wires used for the experiments are defined electrostatically in the two-dimensional electron gas (2DEG) of (Al, Ga)As heterostructures. The lay-out of the TiAu gates is given schematically in Fig. 1. We report here on two wires, both fabricated from an (Al, Ga)As wafer with electron density $n = 2.7 \times 10^{11}\,cm^{-2}$ and $l_{imp} = 19.7\,\mu m$. Both wires have a width $W = 4.0\,\mu m$, but they differ a factor of two in length L: $L = 63.7\,\mu m$ in one wire, and $L = 127.3\,\mu m$ in the other. For transport measurements, the samples are kept in a cryostat at 1.5 K, and at zero magnetic field. The differential resistance is measured with standard low-frequency lock-in techniques, using a $100\,\mu V$ a.c. voltage.

In order to be able to study the effects of NEES separately from electron–phonon scattering effects, we utilize a peculiarity of (Al, Ga)As 2DEGs at low temperatures. In these materials, the coupling between hot electrons and the lattice is orders of magnitude smaller than the coupling within the electron system. This allows one to achieve selective Joule heating of the electron gas in the wires by passing a d.c. current I through the wire. This technique has proven to be very very useful for the study of thermoelectric phenomena in nanostructures[8]. The wires studied here are equipped with opposing pairs of point contacts in their boundaries, allowing us to determine the electron temperature T as a function of

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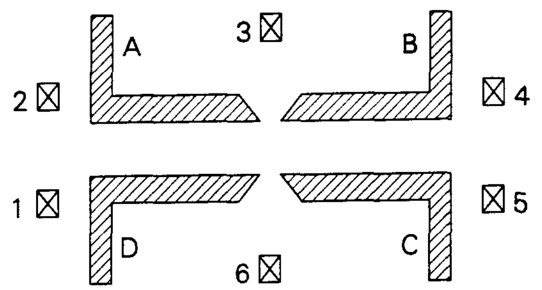

Fig. 1. Lay-out of the gates defining the wires used in the experiments. Current is passed between Ohmic contacts 2 and 4, and the voltage drop is measured between contacts 1 and 5. Ohmic contacts 3 and 6 can be used for measuring the thermovoltages across the point contacts in the wire boundaries. The wires studied here both have a width $W = 4\,\mu$, but differ in length ($L = 63.7$ and $127.3\,\mu$m).

I, using the quantized thermopower of a point contact, as described in Ref. [9]. A typical example of such a measurement of T vs I is shown in Fig. 2. We find that for $|I| \leqslant 20\,\mu$A, and a lattice temperature $T_l \leqslant 2$ K, the electron temperature T in our wires is approximately given by:

$$T = T_l + (I/W)^2 \rho C, \tag{1}$$

where ρ is the resistivity of the channel. The constant C is of order $C \approx 0.05\,\mathrm{m^2K/W}$.

In Fig. 3 we show our data (drawn lines) on the differential resistance ($\mathrm{d}V/\mathrm{d}I$) of both wires. The top trace was obtained from the longer, the bottom trace from the shorter wire. For both wires we observe a remarkable behaviour of $\mathrm{d}V/\mathrm{d}I$: an initial increase, followed by a *decrease* in $\mathrm{d}V/\mathrm{d}I$ with increasing $|I|$[10].

To see whether NEES could in principle be responsible for this behaviour, let us estimate the electron–electron scattering mean-free-path l_{ee} at $T \approx 13$ K [which, according to eqn (1), is the electron temperature in the wire at $I = 15\,\mu$A and $T_l = 1.5$ K]. We have $l_{ee} = v_F \tau_{ee}$, where v_F is the Fermi velocity,

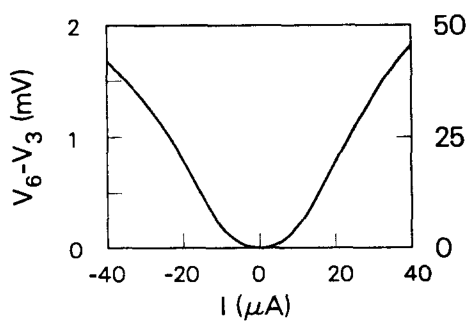

Fig. 2. Transverse voltage $V_6 - V_3$ as a function of heating current I. In this experiment, point contact AB is adjusted between the $N = 1$ and $N = 2$ for maximum quantized thermopower, and the thermopower of point contact CD can be neglected. The electron temperature in the channel can now be deduced from the size of the transverse voltage. See Refs. [8(b),9] for details.

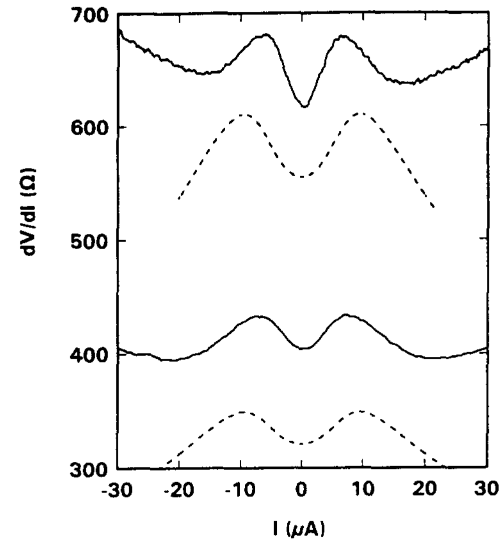

Fig. 3. Drawn curves: experimental dependence of $\mathrm{d}V/\mathrm{d}I$ on heating current i for the $L = 127.3\,\mu$m (top curve) and $L = 63.7\,\mu$m wire (bottom curve), respectively. The dashed curves are the result of our calculations, obtained using boundary scattering parameter α is 0.7, and width $W = l_{imp}/5.5$.

and τ_{ee} the electron–electron scattering time, given by[11]:

$$\frac{1}{\tau_{ee}(T)} = \frac{E_F}{h}\left(\frac{k_B T}{E_F}\right)^2 \left[\ln\left(\frac{E_F}{k_B T}\right) + \ln\left(\frac{2q}{k_F}\right) + 1\right]. \tag{2}$$

Here q is the 2D Thomas–Fermi screening wave-vector ($q = me^2/2\pi\epsilon_r\epsilon_0\hbar^2$). We find $l_{ee} \approx 1.7\,\mu$m, which is much smaller than W. In this limit, the electrons undergo a random-motion due to frequent NEES events, and we assign the decrease in $\mathrm{d}V/\mathrm{d}I$ to the Gurzhi effect. For currents below $8\,\mu$A, $\mathrm{d}V/\mathrm{d}I$ is positive. As $l_{ee} \gtrsim W$ for $|I| = 8\,\mu$A and $T_l = 1.5$ K, this additional feature occurs in the right current range for the electronic Knudsen effect. Moreover, we see that the total increase in $\mathrm{d}V/\mathrm{d}I$ in the long wire is twice the increase in the short wire. This proportionality to L rules out a contact-resistance effect as an explanation for the anomalies.

In order to substantiate our assignment of the anomalous behaviour of $\mathrm{d}V/\mathrm{d}I$ to hydrodynamic phenomena, we have performed model calculations of the effect of NEES on the differential resistance of a two-dimensional wire. We have included NEES events in an electron path-tracing method originally due to Chambers[12, 5], and obtained a solution for arbitrary l_{ee}, l_{imp} and W. In this method one follows the path of an electron until it relaxes at an impurity or the boundary of the system, and then determines a weighted average of the path lengths. The resulting effective mean free path l_{eff} is related to the resistivity ρ of the wire by:

$$\rho^{-1} = \frac{ne^2}{mv_F} l_{eff}. \tag{3}$$

We assume that a fraction p of the incident electrons is reflected specularly at the boundary, the remainder being scattered diffusively, and obtain for the effective mean free path at position x along the width of the wire:

$$l_{\mathrm{eff}}(x) = l - \frac{4l}{\pi}\int_0^1 \mathrm{d}u\sqrt{1-u^2}\,\frac{(1-p)\mathrm{e}^{-x/lu}}{1-p\mathrm{e}^{-W/lu}} + \frac{2}{\pi l_{\mathrm{ee}}}\int_0^1 \mathrm{d}u\,\frac{\sqrt{1-u^2}}{u}\int_0^W \mathrm{d}x'[l_{\mathrm{eff}}(x') + l_{\mathrm{eff}}(W-x')]\left[\mathrm{e}^{-(x-x')/lu}\Theta(x-x') + \frac{p\mathrm{e}^{-(x+x')/lu}}{1-p\mathrm{e}^{-W/lu}}\right], \quad (4)$$

where $l^{-1} \equiv l_{\mathrm{imp}}^{-1} + l_{\mathrm{ee}}^{-1}$, and $\Theta(x)$ is the unit-step function. The average effective mean free path can now be obtained from $l_{\mathrm{eff}} = (1/W)\int_0^W \mathrm{d}x l_{\mathrm{eff}}(x)$. Equation (4) is solved self-consistently using numerical methods.

For a comparison with the experiments, we relate l_{ee} to I using eqns (1)–(3). The resistance of the wire is obtained from $R \equiv V/I = h\pi/2e^2k_{\mathrm{F}}W + \rho L/W$, where the first term is the two-dimensional Sharvin contact-resistance[13]. Subsequently, $\mathrm{d}V/\mathrm{d}I$ is evaluated numerically. The dotted lines in Fig. 3 are the results of our calculation. In both cases, the calculated $\mathrm{d}V/\mathrm{d}I$ values are 60–80 Ω smaller than the experimental values. This is due to the resistance of the wide 2DEG leading to the wires, which is not included in the calculations. In addition, we did not include the lattice heating for currents $|I| > 20\,\mu$A in our modelling. Apart from this the agreement between experiment and theory is very good. Two remarks must be made regarding the parameters used in our modeling. Firstly, one expects that due to depletion the electronic width of the wires is slightly smaller than the lithographic width; we have set $W = l_{\mathrm{imp}}/5.5$ for both wires. In addition, we noticed that using a constant value of p for all angles of incidence leads either to a too large value for $\mathrm{d}V/\mathrm{d}I$ at zero heating current, or a too small Knudsen effect. It is well known from metal wires that in reality p depends on the angle of incidence θ, such that $p \to 1$ for grazing incidence ($\theta \to \pm\pi/2$. According to Ref. [14]:

$$p(\theta) = \exp[-(\alpha\cos\theta)^2]. \quad (5)$$

Using this expression in eqn (4)—where $u = \cos\theta$—we obtain the numerical results of Fig. 3. Good agreement with the experimental data is found with $\alpha = 0.7$, which implies that some 80% of all boundary collisions are specular. A high specularity for boundary scattering in split-gate wires was previously found in magnetoresistance experiments[15].

In summary, we have found convincing evidence of the occurrence of electronic Knudsen and Poiseuille transport regimes in the non-linear differential resistance of split-gate defined wires in a 2DEG. Our results verify speculations on hydrodynamic flow phenomena in solids that date back to the 1950's and 60's, which only came within reach of the experimentalist after the development of metals of sufficiently high mobility, and nano-lithography techniques.

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