

A Review of Instanton Quarks and Confinement

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Abstract. We review the recent progress made in understanding instantons at finite temperature (calorons) with non-trivial holonomy, and their monopole constituents as relevant degrees of freedom for the confined phase.

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INTRODUCTION

New instantons (also called calorons) have been obtained recently, where the Polyakov loop at spatial infinity (the so-called holonomy) is non-trivial [1, 2]. Trivial holonomy, i.e. with values in the center of the gauge group, is typical for the deconfined phase [3, 4]. Non-trivial holonomy is therefore expected to play a role in the confined phase (i.e. for $T < T_c$) where the trace of the Polyakov loop fluctuates around small values.

The Polyakov loop plays the role of the Higgs field, $P(t, \vec{x}) = P \exp \left(\int_0^\beta A_0(t+s, \vec{x}) ds \right)$, where $\beta = 1/kT$ is the period in the imaginary time direction. For $SU(n)$, finite action requires this to tend to

$$\mathcal{P}_\infty = \lim_{|\vec{x}| \rightarrow \infty} P(0, \vec{x}) = g^\dagger \exp(2\pi i \text{diag}(\mu_1, \mu_2, \dots, \mu_n)) g, \quad (1)$$

where g is chosen to bring \mathcal{P}_∞ to its diagonal form, with the n eigenvalues being ordered according to $\sum_{i=1}^n \mu_i = 0$ and $\mu_1 \leq \mu_2 \leq \dots \leq \mu_n \leq \mu_{n+1} \equiv 1 + \mu_1$. One can recognize $8\pi^2 v_m / \beta$ (with $v_m = \mu_{m+1} - \mu_m$) as being the monopole mass.

Monopoles as constituents are close to the picture of instanton quarks, which was already introduced more than 25 years ago [5]. The only difference is that instanton quarks were pointlike, whereas here we have to work in terms of monopole degrees of freedom. We will investigate in how far this plays a role in describing confinement.

Caloron solutions are such that the total magnetic charge vanishes. The "force" stability of these solutions in terms of its constituent monopoles is based, as for exact BPS multi-monopole solutions, on balancing the electromagnetic with the scalar (Higgs) force [6], except that for calorons repulsive and attractive forces are interchanged as compared to multi-monopoles. A single caloron with topological charge one contains $n - 1$ monopoles with a unit magnetic charge in the i -th $U(1)$ subgroup, which are compensated by the n -th monopole of so-called type $(1, 1, \dots, 1)$, having a magnetic charge in each of these subgroups. At topological charge k there are kn constituents,

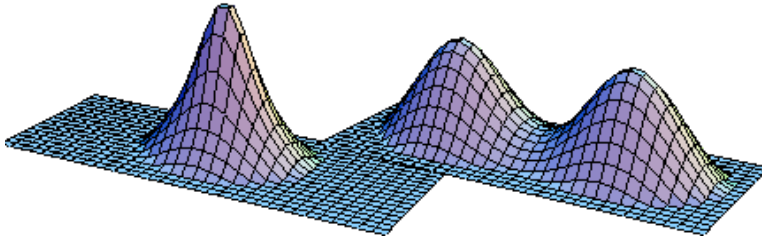


FIGURE 1. Profiles for a caloron at $\omega = 1/4$ with $\rho \ll T$ (left) and $\rho = T$ (right), where vertically the logarithm of the action density is plotted, cutoff below $1/(2e)$.

k monopoles of each of the n types. The sum rule $\sum_{j=1}^n v_j = 1$ guarantees the correct action, $8\pi^2 k$, for calorons with topological charge k .

ONE-LOOP CORRECTIONS

Prior to their explicit construction, calorons with non-trivial holonomy were considered irrelevant [4], because the one-loop correction gives rise to an infinite action barrier. However, the infinity simply arises due to the integration over the finite energy density induced by the perturbative fluctuations in the background of a non-trivial Polyakov loop [7]. The non-perturbative contribution of calorons (with a given asymptotic value of the Polyakov loop) to this energy density as the relevant quantity to be considered, was first calculated in supersymmetric theories [8], where the perturbative contribution vanishes. It has a minimum where the trace of the Polyakov loop vanishes, i.e. at maximal non-trivial holonomy. Recently the calculation of the non-perturbative contribution was performed in ordinary gauge theory at high temperatures [9]. When added to the perturbative contribution with its minima at center elements, these minima turn unstable for decreasing temperature right around the expected value of T_c . This lends some support to monopole constituents being the relevant degrees of freedom which drive the transition from a phase in which the center symmetry is broken at high temperatures to one in which the center symmetry is restored at low temperatures.

A CALORON GAS MODEL FOR CONFINEMENT

A caloron gas model has been constructed recently for SU(2) [10], where one solves for overlapping instantons approximately. One takes

$$A_\mu^{\text{per}}(x) = e^{-2\pi i \vec{\omega} \cdot \vec{\tau}} \sum_i A_\mu^{(i), \text{alg}}(x) e^{2\pi i \vec{\omega} \cdot \vec{\tau}} + 2\pi \vec{\omega} \cdot \vec{\tau} \delta_{\mu 4} \quad (2)$$

to be valid when the density is of the order of 1 fm^{-4} and size ρ is roughly 0.33 fm . In other words, one adds the caloron gauge fields (with the *same* $\mathcal{P}_\infty = e^{2\pi i \vec{\omega} \cdot \vec{\tau}}$) in the algebraic gauge $A_\mu^{\text{alg}}(x + \beta) = \mathcal{P}_\infty A_\mu^{\text{alg}}(x) \mathcal{P}_\infty^{-1}$ in order not to change the boundary

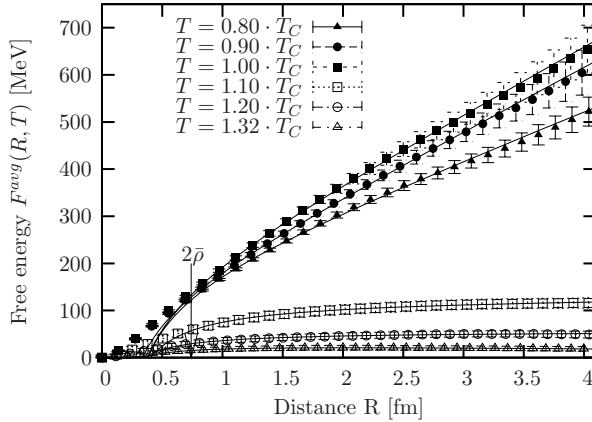


FIGURE 2. Free energy versus distance R at different temperatures $T/T_C = 0.8, 0.9, 1.0$ for the confined and at $T/T_C = 1.10, 1.20, 1.32$ for the deconfined phase in the fundamental representation.

conditions. Only at the end one transforms to the periodic gauge. This has been shown to be exact for multi-calorons [11], but for the above parameters it is a good approximation for a superposition of (anti)calorons.

Remarkably this seems to give confinement for $T < T_c$ and deconfinement for $T > T_c$. In the confining phase one imposes $\omega = |\vec{\omega}| = 1/4$ and $\text{Tr} \mathcal{P}_\infty = 0$, whereas in the deconfining phase one tends to find $\omega = 0$ (or $1/2$) and $\text{Tr} \mathcal{P}_\infty = 2$ (one takes into account that ω only gradually becomes 0 or $1/2$ with increasing temperature, but we will ignore this here). In figure 1 the caloron is shown for $\omega = 1/4$, where we contrast $\rho \ll T$ and $\rho = T$. Of course ρ is somewhere in between, but it clearly gives a confining force over the distances probed.

To show this they have solved for

$$D_1(\rho, T) = A_1 \rho^{b-5} \exp(-c\rho^2) \quad \text{and} \quad D_2(\rho, T) = A_2 \rho^{b-5} \exp(-4[\pi\rho T]^2/3), \quad (3)$$

where in the first case $\bar{\rho}$ is fixed, $T \leq T_c$ and $\omega = 1/4$ (which means $v = 1/2$), and in the second case $\bar{\rho}$ is running, $T \geq T_c$ and $\omega = v = 0$. Finally one requires $\bar{\rho}(T_c)_{\text{conf}} = \bar{\rho}(T_c)_{\text{deconf}} = 0.37 \text{ fm}$, which determines c . With $b = (11n - 2n_f)/3 = 22/3$ ($n_f = 0$) and $\int D_{1,2}(\rho, T) d\rho = 1$ this gives the model. Determining $\bar{\rho}(T < T_c)$, they have also fixed $T_c \approx 178 \text{ MeV}$ and $\sigma(0) \approx 318 \text{ MeV/fm}$.

In figure 2 the free energy versus the distances at different temperatures is given and although the string tension should go to zero as one approaches T_c from below, it is true that for $T < T_c$ the string tension is finite and becomes zero for $T > T_c$. This model, in a sense, assumes weak coupling. Also in the spatial Wilson loops one finds an area law.

DENSE MATTER

There has been yet another development that introduces instanton quarks to describe confinement [12], which has been summarized in [13]. At low energies and large chem-

ical potential the η' interactions are determined by ordinary instantons, with a periodicity of θ which is 2π . But at small chemical potential (and temperature) one finds for $\eta' = \phi = \text{Tr}(U)$, where U is the chiral matrix, that (ignoring the mass corrections)

$$L_{\eta'} = f^2(\partial_\mu \phi)^2 + \lambda \cos([\phi - \theta]/n). \quad (4)$$

Now the topological charge is $Q_a = \pm 1/n$, but with the sum $Q = \sum_a Q_a$ an integer. The conjecture is that in the confined phase instanton quarks can be far apart, but remain strongly correlated, requiring large and overlapping instantons. One has to see if it is strongly interacting and if the constituents are line like (the constituent monopoles), instead of point like (at least semi-classically). The conclusions are nevertheless interesting.

In conclusion instanton quarks seem to play a role in the confined phase. The interpretation is of course different than what was assumed in [5], where now the time coordinate is replaced in a sense by a phase. What remains true is, however, that charge k $\text{SU}(n)$ solutions are described by kn lumps of charge $1/n$.

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