A test of general relativity using radio links with the Cassini spacecraft

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According to general relativity, photons are deflected and delayed by the curvature of space-time produced by any mass[1–3]. The bending and delay are proportional to $\gamma + 1$, where the parameter γ is unity in general relativity but zero in the newtonian model of gravity. The quantity $\gamma - 1$ measures the degree to which gravity is not a purely geometric effect and is affected by other fields; such fields may have strongly influenced the early Universe, but would have now weakened so as to produce tiny—but still detectable—effects. Several experiments have confirmed to an accuracy of ~0.1% the predictions for the deflection[4,5] and delay[6] of photons produced by the Sun. Here we report a measurement of the frequency shift of radio photons to and from the Cassini spacecraft as they passed near the Sun. Our result, $\gamma = 1 + (2.1 \pm 2.3) \times 10^{-5}$, agrees with the predictions of standard general relativity with a sensitivity that approaches the level at which, theoretically, deviations are expected in some cosmological models[7,8].

Testing theories of gravity in the Solar System and with binary pulsars has been pursued for a long time[1,2], yet general relativity has survived whereas most of its alternatives have been disproved. In particular, the other main test—the anomalous advance of the pericentre of an orbiting body, such as Mercury around the Sun—has been found in agreement with Einstein's prediction, with a similar accuracy ~0.1%. In the past 20 yr there has been no appreciable improvement. With the Cassini mission, this barrier has now been largely overcome as far as γ is concerned, but no violations of general relativity have been detected.

The increase Δt produced by the gravitational field of the Sun (with mass M_S and radius R_S) in the time taken for light to travel the round trip between the ground antenna and the spacecraft, at distances r_1 and r_2 respectively from the Sun, is[1]:

$$\Delta t = 2(1+\gamma)\frac{GM_S}{c^3}\ln\left(\frac{4r_1 r_2}{b^2}\right) \tag{1}$$

where G is the gravitational constant, b ($\ll r_1$, r_2) the impact parameter and c the velocity of light. The motion of the spacecraft and Earth produces a change in b and Δt, equivalent to a change in distance, and hence a change in relative radial velocity. The corresponding fractional frequency ($y_{gr} = \Delta\nu/\nu$) shift for a two-way radio signal is[9]:

$$y_{gr} = \frac{d\Delta t}{dt} = -2(1+\gamma)\frac{GM_S}{c^3 b}\frac{db}{dt} = -(1\times10^{-5}\,\mathrm{s})(1+\gamma)\frac{1}{b}\frac{db}{dt} \tag{2}$$

For a spacecraft much farther away from the Sun than the Earth, db/dt is not very different from the Earth's velocity $v_E = 30\,\mathrm{km\,s^{-1}}$. In the Cassini solar conjunction the peak value of y_{gr} is 6×10^{-10}. The Cassini experiment, exploiting the new observable y_{gr} (refs 9, 10), was carried out between 6 June to 7 July 2002, when the spacecraft was on its way to Saturn, around the time of a solar conjunction (Fig. 1). The gravitational signal and the tracking passes that provided useful data are shown in Fig. 2.

The main reason why the Doppler method has not been applied before is the overwhelming noise contribution due to the solar corona. The Cassini mission has overcome this hindrance with: (1) high-frequency carrier waves in the Ka-band, in addition to the X-band for standard operation; and (2) a multi-frequency link in which three different phases are measured at the ground station[11,12]. Two carriers at 7,175 MHz (X-band) and 34,316 MHz (Ka-band) are transmitted from the ground; whereas, in addition to the downlink carriers at 8,425 MHz and 32,028 MHz locked on board to the X and the Ka signals respectively, a nearby Ka-band downlink carrier coherent with the X-band uplink is also transmitted back. This novel radio configuration uses dedicated and advanced instrumentation, both on board the spacecraft and at the ground antenna, and allows a full cancellation of the solar plasma noise (see Supplementary Fig. S1 for details)[13–15]. The resulting measurement errors are four orders of magnitude smaller than the relativistic signal in equation (2).

The new ground station DSS25 at the NASA Deep Space Network complex in Goldstone, California, has performed admirably, par-

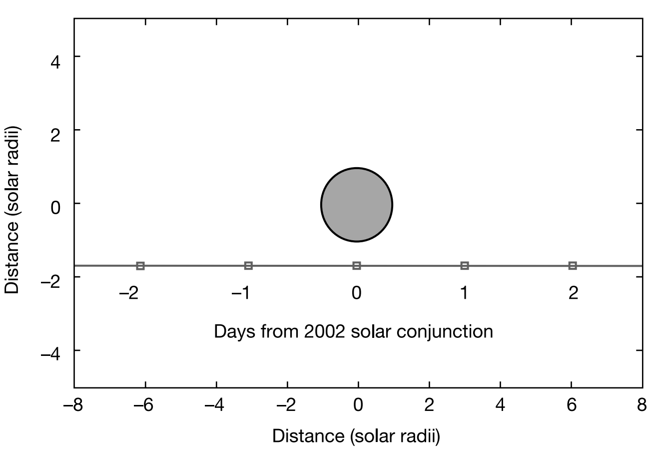

Figure 1 Geometry of the 2002 Cassini solar conjunction. The graph shows Cassini's motion in the sky relative to the Sun, as a function of days from the 2002 solar conjunction; coordinates are in solar radii. The conjunction—at which the spacecraft (at a geocentric distance of 8.43 AU), the Sun and the Earth were almost aligned, in this order—occurred on 21 June 2002, with a minimum impact parameter $b_{min} = 1.6\,R_S$, and no occultation.

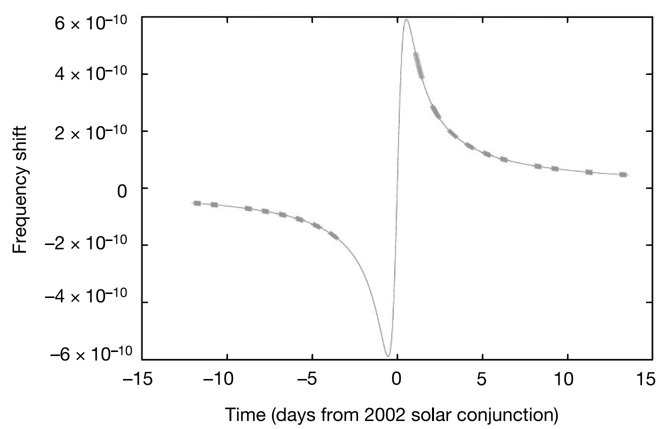

Figure 2 The gravitational signal. The two-way relativistic frequency shift y_{gr} due to the Sun and the available 18 passages, each lasting about 8 h, is shown. Unfortunately, no data could be acquired for three days just before conjunction owing to a failure of the ground transmitter; moreover, the tracking data acquired near closest approach were particularly noisy. A much larger plasma noise was detected in some passes after conjunction, and it was fully removed by the multi-link technique. Remarkably, during this time period, SOHO observations revealed large coronal mass ejections traversing the radio beam.

ticularly with respect to its Ka-band capabilities and special instrumentation designed to measure variations in path delay due to the wet troposphere[13,15]. A Ka–Ka frequency translator (the crucial onboard instrument, provided by the Italian Space Agency) has assured very high phase coherence between the uplink and downlink signals.

The solar corona adds to each carrier very large frequency fluctuations, impairing the accuracy of conventional closed-loop receivers when the ray path is very close to the Sun (within about 10 solar radii); for this experiment, a wide-band, open-loop receiver has been used instead. A specially designed digital frequency estimator[14] has been applied to the received signal (sampled at 1 kHz) to obtain the three required time series $y_{XX}(t)$, $y_{KK}(t)$, $y_{XK}(t)$ of the reconstructed sky frequencies with a resolution of 1 s, where the subscripts X and K correspond to the X-band and Ka-band, respectively.

The final processing of the plasma-free, non-dispersive fractional frequency shift $y_{nd}(t)$ requires the calibration of the dry and wet (water vapour) components of the Earth's troposphere. Its dry part can be determined from ground meteorological data and suitable modelling of elevation-dependent effects. In terms of fractional frequency shift, its magnitude can be as large as 10^{-12}. The wet component is much smaller ($\sim 3 \times 10^{-14}$, over timescales of 1,000–10,000 s) but much more difficult to model. Its contribution was measured at the ground station with especially designed radiometers that are able to measure the water vapour content along the line of sight. In this way the estimated total tropospheric contribution to the optical path variation could be subtracted from the signal, with a very significant improvement of the results.

An important contribution to the relative frequency shift is due to non-gravitational forces acting on the spacecraft. During the experiment the spacecraft was kept in a stable dynamical and thermal state, with its attitude controlled only by momentum wheels, and without turning on or off any subsystems or instruments. No unexpected buffeting or acceleration was detected throughout the experiment. Thanks to its large mass (5,200 kg) and careful design, Cassini turned out to be an extremely stable platform, subject only to small and steady non-gravitational accelerations owing to the solar radiation pressure and the anisotropic thermal emission of the spacecraft (largely produced by three on-board radioisotope thermoelectric generators (RTGs)). The RTGs dissipate about 10 kW of constant thermal power, in large part isotropically radiated. The non-isotropic part of this thermal emission produces an acceleration, whose components are constant in the spacecraft reference frame. Deriving this acceleration from a model of the spacecraft is a difficult task; but its estimation from Doppler measurements, combined with attitude data, is routinely carried out for spacecraft navigation with good and consistent accuracies. The largest component (along the Earth–spacecraft direction, pushing towards the Earth) is about $3 \times 10^{-9}\,\mathrm{m\,s^{-2}}$, more than four orders of magnitude smaller than the peak value of the relativistic acceleration signal $y_{gr}c(db/dt)/b \approx 3.6 \times 10^{-5}\,\mathrm{m\,s^{-2}}$ (corresponding to a velocity change $y_{gr}c$ in the timescale $b/(db/dt)$). This component has been determined with a formal error of $\sim$3%. The other two components (orthogonal to the orbital plane and in the orbital plane) do not significantly affect the orbit and are smaller (1×10^{-10} and $4 \times 10^{-10}\,\mathrm{m\,s^{-2}}$, respectively)—they are determined much less accurately (respectively to 100% and 50%). A better estimation of these components was possible using data from another radio science experiment carried out by Cassini at solar opposition, from 26 November 2002 to 4 January 2003, under similar conditions.

As a consequence of the large distance from the Sun and the small area-to-mass ratio of the spacecraft, the solar radiation produces an acceleration of about an order of magnitude smaller than the RTGs thermal thrust[10]. (The incident solar power is just 175 W, as compared with 10 kW for the RTGs.) Modelling this acceleration at conjunction is simple, as the spacecraft is shaded by the large (4-m diameter) parabolic high-gain antenna. In general, the direction of the force depends on the spacecraft attitude, in particular the solar aspect angle between the axis of the antenna and the Sun. For Cassini, this angle changed by only two degrees during the 30-day experiment and almost vanished at conjunction. Therefore, although the force is almost radial, information concerning the spacecraft attitude and trajectory must be used for its determination in the orbital fit. The magnitude of the thrust depends on the thermo-optical properties of the dish, in particular the specular and diffuse reflectivity coefficients. However, as the spacecraft attitude in an inertial frame does not change much, the orbital fit cannot effectively discriminate between them; nonetheless, the estimated values are consistent with those obtained from previous tracking data. The $1/r^2$ variation of the force (about 2.3% of the average

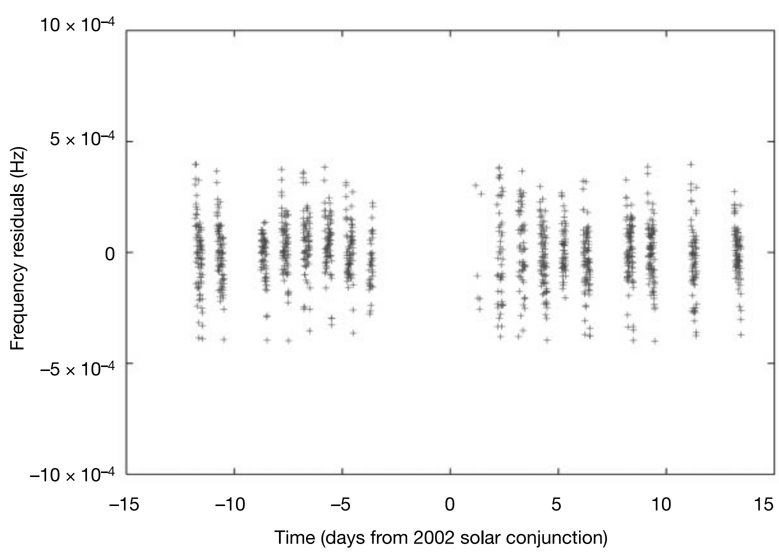

Figure 3 The Doppler residuals. Two-way frequency residuals, relative to an 8.4-GHz carrier, as a function of time (in days from solar conjunction) obtained from the final 9-parameter fit on the calibrated data. The r.m.s. value of the 1,094 data points (compressed at a 300-s integration time) is 1.2×10^{-4} Hz, corresponding to a one-way range rate of $2.2 \times 10^{-6}\,\mathrm{m\,s^{-1}}$; this is unprecedented accuracy for tracking data at solar conjunction.

value, and therefore unappreciable) is, however, accounted for by the model. The variability of the solar constant, and possible small changes in the thermo-optical coefficients due to ageing and temperature variations, are insignificant.

The question at what level can violations of general relativity be expected, does not have a satisfactory answer yet. A long-range scalar field is currently assumed to have a fundamental role in primordial cosmology: although it decays with the expansion of the Universe, its present remnant would entail not only violations of the two main tests of general relativity, but also a lack of universality of the constants of microphysics, as assumed in the equivalence principle[7,8]. A claim, based on quasar absorption lines, that the fine-structure constant α was weaker in the distant past has been made recently, thereby violating the equivalence principle[16–19]. If this claim is confirmed, and reconciled with other constraints on the variation of fundamental constants[20], such a finding would be the first serious challenge to Einstein's model. No detailed theory is available about the expected amounts of these violations, but $\gamma - 1$ should be negative and, possibly, in the range 10^{-5}–10^{-7}. Therefore, our result with accuracy not far from this range places an important constraint on this cosmological scenario. □

Method

The dynamical model used in the orbital fit is particularly simple, thanks to the large distance from the Sun, the location of the spacecraft in interplanetary space and the lack of unknown gravitational perturbations by Solar System bodies. The Jet Propulsion Laboratory's Orbit Determination Program has been used in the integration of the equations of motion and the orbital solution, based on planetary ephemerides and ancillary data, such as station location and Earth orientation parameters. To speed up the data processing, the observables have been compressed at $\tau = 300$ s by differencing the detected phases. Owing to the spectral characteristics of the noise (Supplementary Fig. S2), the data compression does not in any way affect the final result. We have used up to 12 free 'solve-for' parameters: (1) the six components of the state vector at the start of the experiment; (2) the three components of the non-gravitational acceleration due to the RTGs in the spacecraft frame; (3) the specular and diffuse reflectivity of the high-gain antenna, which determine the magnitude (and the direction) of the non-gravitational acceleration owing to solar radiation pressure; (4) the relativistic parameter γ. 'Consider' parameters (quantities not solved-for, but whose uncertainty is taken into account in the solution) include the dry troposphere, the station location, polar motion and the Earth Love numbers (which intervene in the solid tide model).

As discussed above, among the five parameters that control the non-gravitational acceleration, three (the non-radial components of the thermal thrust from the RTGs and one of the two optical coefficients of the high-gain antenna) are poorly determined. It is therefore appropriate to investigate a solution including only the other two, namely the radial acceleration due to the RTGs and the diffuse reflectivity of the antenna; by so doing, most non-gravitational perturbations are accounted for at a level consistent with the accuracy of the tracking data. The value of the other three parameters, the specular reflectivity and the non-radial components of the RTGs acceleration, together with their uncertainties have been taken from a separate fit carried out on the data from the Cassini solar opposition experiment, as mentioned above. This is our main orbital fit, with only nine parameters to be determined. The a priori uncertainty of the parameters that are not estimated, but affect the solution (such as the station geocentric coordinates and those derived from the gravitational wave experiment), has been included in the computation of the covariance matrix.

The data are assumed to be independent, but for each passage they are weighted with their own standard deviation. The variability of the results has been explored with different assumptions, in particular: a change in the threshold for discarding the outliers; different sampling time; and fitting separately the data before and after the conjunction. These trials clarified several issues including the structure of the covariance matrix, but did not in any way mar the final result.

The residuals of the orbital fit (Fig. 3) show a remarkably white spectrum (see Supplementary Fig. S2), which corresponds to the $\sim 1/\sqrt{\tau}$ observed dependence of both the root-mean-square (r.m.s.) deviation and the Allan deviation from the sampling time τ used in the orbital fit. After obvious outliers are removed (mostly introduced by incorrect tropospheric calibrations), the statistical distribution of the data shows a clearly gaussian behaviour.

We have also explored the full 12-parameter orbital fit and obtained similar results, with $\gamma = 1 + (1.35 \pm 2.47) \times 10^{-5}$, and no appreciable variation of the r.m.s. value of the residuals. Reassuringly, the non-gravitational accelerations so obtained are also fully consistent with the value obtained from the previous opposition experiment and with a long arc orbital solution generated by the Cassini Navigation Team using a data set spanning almost two years. In another trial, in addition to the spacecraft state vector we used just two free parameters—γ and a single, radial non-gravitational acceleration—and obtained the result $\gamma = 1 + (0.21 \pm 2.43) \times 10^{-5}$, with a small increase in the r.m.s. value of the residuals. The understanding of the physics involved and the number of different checks that have been performed have increased our confidence in the results.

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1. Will, C. M. *Theory and Experiment in Gravitational Physics* (Cambridge Univ. Press, Cambridge, 1993).
2. Will, C. M. The confrontation between General Relativity and experiment. *Living Rev. Relativity* **4,** 4 (cited on 15 Aug 2003); at ⟨http://www.livingreviews.org/lrr-2001-4⟩.
3. Ciufolini, I. & Wheeler, J. A. *Gravitation and Inertia* (Princeton Univ. Press, Princeton, 1995).
4. Dyson, F. W., Eddington, A. S. & Davidson, C. A determination of the deflection of light by the Sun's gravitational field, from observations made at the total eclipse of May 29, 1919. *Phil. Trans. R. Soc. Lond. A* **220,** 291–333 (1920).
5. Lebach, D. E. *et al.* Measurement of the solar gravitational deflection of radio waves using very-long-baseline interferometry. *Phys. Rev. Lett.* **75,** 1439–1442 (1995).
6. Reasenberg, R. D. *et al.* Viking relativity experiment: verification of signal retardation by solar gravity. *Astrophys. J.* **234,** L219–L221 (1979).
7. Damour, T. & Polyakov, A. M. The string dilaton and a least coupling principle. *Nucl. Phys. B* **423,** 532–558 (1994).
8. Damour, T., Piazza, F. & Veneziano, G. Violations of the equivalence principle in a dilaton-runaway scenario. *Phys. Rev. D* **66,** 046007 (2002).
9. Iess, L., Giampieri, G., Anderson, J. D. & Bertotti, B. Doppler measurement of the solar gravitational deflection. *Class. Quant. Grav.* **16,** 1487–1502 (1999).
10. Bertotti, B. & Giampieri, G. Relativistic effects for Doppler measurements near solar conjunction. *Class. Quant. Grav.* **9,** 777–793 (1992).
11. Bertotti, B., Comoretto, G. & Iess, L. Doppler tracking of spacecraft with multifrequency links. *Astron. Astrophys.* **269,** 608–616 (1993).
12. Bertotti, B. & Giampieri, G. Solar coronal plasma in Doppler measurements. *Solar Phys.* **178,** 523–545 (1998).
13. Iess, L. *et al.* in *Aerospace Conf. Proc. (Montana, March 2003)* Vol. 1, 205–211 (Institute of Electrical and Electronics Engineers (IEEE), 2003).
14. Tortora, P., Iess, L. & Ekelund, J. E. in *Proc. World Space Congr. (Houston, 2002)* Paper IAC-02-Q.2.03 (AIAA, Reston, VA, 2002).
15. Tortora, P., Iess, L. & Herrera, R. G. in *Aerospace Conf. Proc. (Montana, March 2003)* Vol. 3, 1465–1473 (Institute of Electrical and Electronics Engineers (IEEE), 2003).
16. Murphy, M. T. *et al.* Possible evidence for a variable fine-structure constant from QSO absorption lines: motivations, analysis and results. *Mon. Not. R. Astron. Soc.* **327,** 1208–1222 (2001).
17. Murphy, M. T. *et al.* Possible evidence for a variable fine-structure constant from QSO absorption lines: systematic errors. *Mon. Not. R. Astron. Soc.* **327,** 1223–1236 (2001).
18. Murphy, M. T. *et al.* Further constraints on variation of the fine-structure constant from alkali-doublet QSO absorption lines. *Mon. Not. R. Astron. Soc.* **327,** 1237–1243 (2001).
19. Murphy, M. T. *et al.* Improved constraints on possible variations of physical constants from HI 21-cm and molecular QSO absorption lines. *Mon. Not. R. Astron. Soc.* **327,** 1244–1248 (2001).
20. Damour, T. String theory, cosmology and varying constants. *Astrophys. Space Sci.* **283,** 445–456 (2003).

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Uniform resonant chaotic mixing in fluid flows

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Laminar flows can produce particle trajectories that are chaotic[1,2], with nearby tracers separating exponentially in time. For time-periodic, two-dimensional flows and steady three-dimensional (3D) flows, enhancements in mixing due to chaotic advection are typically limited by impenetrable transport barriers that form at the boundaries between ordered and chaotic mixing regions. However, for time-dependent 3D flows, it has been proposed theoretically[3–5] that completely uniform mixing is possible through a resonant mechanism[5] called singularity-