The Einstein-De Sitter Debate and Its Aftermath

Michel Janssen[1]

I

The recently published Vol. 8 of Einstein's *Collected Papers* brings together for the first time all extant letters and postcards documenting the famous debate of 1916–18 between Einstein and the Leyden astronomer Willem de Sitter (1872–1934), over, as they referred to it, the relativity of inertia. It was in the course of this debate that the first two relativistic cosmological models were proposed: the "Einstein cylinder world," filled with a uniform static mass distribution; and the completely empty "De Sitter hyperboloid world" (a name introduced in *Weyl 1923*, p. 293). In discussing the latter, Einstein and De Sitter had difficulty distinguishing features of the model from artifacts of its various coordinate representations. The situation was clarified in 1918 in correspondence between Einstein and two of the greatest mathematicians of the era, Hermann Weyl (1885–1955) in Zurich and Felix Klein (1849–1925) in Göttingen. (Thanks to Klein and David Hilbert (1862–1943), Göttingen in those days was the math capital of the world.) Some of the issues that Einstein discussed with De Sitter also come up in Einstein's correspondence in 1918 with the German physicist Gustav Mie (1868–1957).

The picture that emerges is one of Einstein holding on with great tenacity to two beliefs concerning the universe that guided him in the construction of his cosmological model: first, that the universe is static; and second, that its metric structure is fully determined by matter—in other words, that its metric field[2] satisfies what, in 1918, he called "Mach's principle." De Sitter's vacuum solution of Einstein's field equations with cosmological term is a counterexample to this principle, and, for this reason, Einstein tried to discard it on various grounds. Two main lines of attack can be discerned: one was to argue that the De Sitter solution is not static; the other was to argue that it has what today would be called an intrinsic singularity,[3] which in turn was used to argue that it is not matter-free. In the end, Einstein had to acknowledge that the solution is fully regular and matter-free and hence indeed a counterexample to Mach's principle, but he could still discard the solution as physically irrelevant because it is not globally static.

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1. Secs. I–III of this handout are based on the editorial note, "The Einstein-De Sitter-Weyl-Klein Debate," in Vol. 8 of *The Collected Papers of Albert Einstein*, covering correspondence during period 1914–1918 (*Schulmann et al. 1998*, pp. 351–357).
2. The following analogy should suffice to understand the notion of a *metric field* well enough to read this handout. Consider a map of the earth (which is essentially a way of coordinatizing the globe). We cannot simply take distances on the map (the coordinate distances) to represent distances on the globe (the actual or proper distances). For instance, a horizontal line segment of two inches on the map near the equator will correspond to a larger distance on the globe than a horizontal line segment of two inches on the map near the poles. For every point on the map, we need to specify a set of numbers with which we have to multiply distances on the map in the vicinity of that point (*coordinate distances*) to convert them to actual distances (*proper distances*). (It will be clear that we need more than one number because the conversion for north-south distances will be different from the conversion for east-west distances.) The numbers in such a set are called the *components of the metric* at that point. The metric *field* is the collection of all such sets of numbers for all points on the map. The same thing we do here with 2-dim. space (representing the 2-dim. curved surface of the earth on a 2-dim. Euclidean plane together with a specification of the metric field to do all conversion from coordinate distances to proper distances) we can do with 4-dim. space-time as well. There will now also be a number (the temporal component of the metric) by which we have to multiply coordinate time differences to convert them to proper time differences.

II

The debate between Einstein and De Sitter began in the fall of 1916. This can be inferred from De Sitter's references, in the first letter of the debate of November 1, 1916 and in *De Sitter 1916a*, to conversations that they had during a recent visit of Einstein to the Netherlands. They agreed that general relativity, as it stood, preserved a remnant of Newton's absolute space and time, since boundary conditions played a role alongside matter in determining the metric field and thereby the inertial properties of the universe. De Sitter did not find this at all objectionable, but Einstein wanted to eliminate this absolute element by postulating degenerate values for the metric field at infinity—which he thought would ensure that the inertial mass of test particles at infinity vanishes (see *Einstein 1917*, pp. 145–146)—and the existence of distant masses that would somehow cause these degenerate values at infinity to turn into Minkowskian values at large but finite distances. De Sitter sharply criticized this proposal. He argued that Einstein's distant masses would have to be outside the visible part of the universe, and that an explanation of the origin of inertia invoking such invisible masses was no more satisfactory than one invoking Newton's absolute space and time.

Einstein came to accept De Sitter's criticism and abandoned the proposal. As he wrote to De Sitter on February 2, 1917: "I have completely abandoned my views, rightfully contested by you, on the degeneration of the $g_{\mu\nu}$. I am curious to hear what you will have to say about the somewhat crazy idea I am considering now." In his famous paper "Cosmological Considerations on the General Theory of Relativity" (*Einstein 1917*, see *Lightman 1991*, p. 16) published later that month, he circumvented the problem of boundary conditions at infinity simply by abolishing infinity! That is to say, he introduced a spatially closed model of the universe. A new term involving the so-called cosmological constant had to be added to the field equations to allow this model as a solution. As he emphasized in the final paragraph of the paper, however, the new term is needed not so much to allow for a closed universe as to allow for a closed *static* universe (*Einstein 1917*, p. 152). Writing to De Sitter in early March, Einstein was careful to emphasize that his model was intended primarily to settle the question "whether the basic idea of relativity can be followed through to its conclusion, or whether it leads to contradictions," and that whether the model corresponds to reality was another matter. Nevertheless, in the next paragraph of the same letter, he explored some physical consequences of the model.

As De Sitter went over Einstein's calculations, he discovered another cosmological model. In Einstein's model, the spatial geometry is that of a 3-dimensional hypersphere embedded in a 4-dimensional Euclidean space. Using an imaginary time coordinate, De Sitter considered an alternate model, in which the space-*time* geometry is that of a 4-dimensional hypersphere embedded in a 5-dimensional Euclidean space, or, if the imaginary time coordinate is replaced by a real time coordinate, a 4-dimensional hyper-hyperboloid in a 4+1-dimensional Minkowski space-time.[4]

3. A singularity here means that some component of the metric becomes either infinite or zero at some point. This can mean two things. (1) There is an *intrinsic singularity* at that point, reflecting a pathology of *the space-time itself* (these are the type of singularities you have at the center of a black hole or at the origin of the universe in the standard big bang model). (2) There is a *coordinate singularity.* The singular behavior reflects a pathology of *the map of the space-time*. Consider the analogy with a map of the earth (see the preceding footnote). The north pole itself will actually be represented by a horizontal line at the top of the map. Any segment of that line corresponds to an actual distance of zero. Hence, the relevant component of the metric will vanish for points corresponding to the north pole (the finite coordinate distances at the top of the map need to be multiplied by zero to get the actual zero distance on the globe).

Whereas Einstein's model is a solution of the new field equations for a uniform static mass distribution, De Sitter's alternate model turns out to be a vacuum solution.

In late March 1917, De Sitter sent Einstein a side-by-side comparison of the two models, which can be seen as a draft of the paper that he published shortly afterward in the Proceedings of the Amsterdam Academy of Sciences (*De Sitter 1917a*). Einstein objected to De Sitter's solution because there was no matter producing the curvature of the space-time it described. In his reply of March 24, 1917, Einstein wrote: "It would be unsatisfactory, in my opinion, if a world without matter were possible. Rather, the $g^{\mu\nu}$-field[5] should *be fully determined by matter and not be able to exist without the latter*." Einstein would later introduce the term "Mach's principle" for this requirement (*Einstein 1918a*).

Given this principle, Einstein understandably tried to find fault with De Sitter's solution. His first objection (in that same letter in late March 1917) was that in the so-called stereographic coordinates that De Sitter had used to describe his solution there is a 3-dimensional hyper-hyperboloid on which the values of all components of the metric go to infinity. De Sitter replied that this singular hypersurface corresponds to (temporal) infinity and that the reason the components of the metric become infinite is simply that on this hypersurface finite coordinate distances correspond to infinite proper distances. In Einstein's next (surviving) letter (of June 14, 1917), two new objections were raised to De Sitter's model, namely that it is not static and that it has a preferred center. It is unclear how much weight the first of these objections carried with De Sitter. De Sitter had already objected strongly to Einstein's assumption that on average the matter distribution of the universe is static and homogeneous. As to the second objection, the allegedly preferred center (like the singular hypersurface) turns out to be an artifact of the stereographic coordinates.

Meanwhile, De Sitter began to take his model more seriously as a possible description of the actual universe. Originally, he had looked upon the discussion of the two new models merely as idle speculation about how to extrapolate the approximately Minkowskian values of the metric field beyond the visible part of the universe. By mid-1917, however, he was busy exploring the physical consequences of three different options available for the global structure of space-time (as can be gathered from what he wrote to Einstein on June 20, 1917 and from two papers he published on the topic, *De Sitter 1917b*, *1917c*). He labeled these options A, B, C. They are: (A) Einstein's cylindrical space-time, (B) his own hyperboloid space-time, and (C) Minkowski space-time. De Sitter, like Mie a year later, distinguished between a purely inertial and a gravitational part of the metric field. The three alternatives above pertain to the purely inertial field. For these three cases, De Sitter calculated the gravitational field, i.e., the deviations from the inertial field, produced by a massive spherical body such as the sun. Einstein was pleased that De Sitter was now willing to engage in the sort of speculation that he had rejected earlier. But he took exception to De Sitter's decomposition of the metric field into an inertial and a gravitational part,[6] and, in particular, to the distinction it implied in model (A) between "world matter" generating the inertial field and "ordinary matter" generating the gravitational field. Ordinary matter, Einstein explained in his reply to De Sitter of June 22, 1917, should not be thought of as existing in addition to the uniform mass distribution of his cosmological model, but in terms of local condensa-

4. See below for a picture of this solution when 2 of its 3 spatial dimensions are suppressed. The picture shows a 1+1 dimensional De Sitter space-time embedded in a 2+1 dimensional Minkowski space-time.
5. The standard notation for the metric field is $g_{\mu\nu}$ (pronounce: *g-mu-nu*). In *Surely you are joking, Mr. Feynman*, Richard Feynman relates the story that he once found his way from the airport to the hotel where a general relativity conference was taking place by telling the cab driver: "take me to the place where you took all the other people going '*g-mu-nu g-mu-nu*'."
6. It was probably is part in response to De Sitter and Mie distinguishing between an inertial and a gravitational component of the metric field that Einstein in 1918 gave a new and more satisfactory formulation of his equivalence principle, viz. that inertia and gravity are of the exact same nature.

tions in this uniform mass distribution. The spatial geometry of a more realistic version of his model with local inhomogeneities, he added, might resemble that of the "surface of a potato" more than that of a perfect sphere. Einstein did not develop any such models. De Sitter's calculations, and Mie's a year later, made it doubtful that static inhomogeneous variants of Einstein's model could be constructed. Mie concluded on the basis of his calculations (which turned out to be erroneous) that any inhomogeneous matter distribution in a more realistic variant of Einstein's model would quickly evolve back to the completely homogeneous distribution of the original model.

In 1930, Sir Arthur Stanley Eddington (1882–1944), famous for his role in the eclipse expeditions of 1919 that confirmed general relativity's prediction of the bending of light, showed that the situation is actually just the opposite of what Mie thought and that it is, in fact, much worse for Einstein. Einstein's model is not the stable equilibrium that the universe would never get away from, as Mie believed. The model is unstable. It would fly apart.[7] But in 1917, the year to which we now return, none of this was known.

To facilitate comparison with options (A) and (C), De Sitter had written the line element for his own model (B) in new coordinates. In these coordinates, the components of the metric tensor are time-independent. Einstein told De Sitter that he found this new form "very instructive," and the remainder of the exchange between Einstein and De Sitter during 1917–18 focused on this so-called static form of the De Sitter solution. In this form, the spatial geometry of the De Sitter model is the same as that of Einstein's model, namely that of a hypersphere in a 4-dimensional Euclidean space. Contrary to Einstein's model, however, the temporal component of the static De Sitter metric is variable and vanishes on the "equator" of this hyperspherical space. Einstein argued that such singular behavior of the metric was unacceptable, and suggested that it indicated the presence of matter on the equator. Careful not to make the same mistakes that he had made in his analysis of De Sitter's solution in stereographic coordinates, Einstein convinced himself that the singularity of the metric in static coordinates occurred at a finite proper distance from an arbitrarily chosen point of the space-time and that it was not just an artifact of the coordinates used. The threat that De Sitter's solution posed to Mach's principle thus finally seemed to be removed. In early March 1918, Einstein completed two papers, *Einstein 1918a* introducing Mach's principle, and *Einstein 1918b* presenting the argument outlined above for why the De Sitter solution is not a counterexample to this principle. De Sitter's defense of his solution (in a letter of April 10, 1918 and in *De Sitter 1918*) was that the singular equator cannot be reached from any point in the space-time, despite the fact that the proper distance from any point to the equator is finite. Only a few months later, it became clear that both this peculiar property and the singularity itself are artifacts of the static coordinates used.

III

The discussion of the cosmological models of Einstein and De Sitter continued in Einstein's correspondence with Hermann Weyl and Felix Klein. The central issue in both exchanges was whether or not antipodal points should be identified in spherically symmetric solutions of Einstein's field equations with cosmological term. This issue was also touched upon in the correspondence with De Sitter. The most important results coming out of the discussion with Weyl and Klein, however, concern the interpretation of the singular equator in the static form of the De Sit-

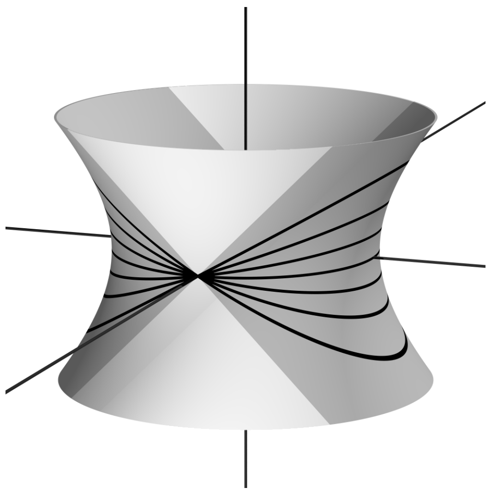
7. Eddington suggested that the actual universe might have started out in a state described by Einstein's model and then developed toward a state described by De Sitter's model.

ter solution.

In the section on cosmology in the first edition of Weyl's famous book *Space-Time-Matter* (*Weyl 1918*), which Einstein read in proof, the author discussed various static spherically symmetric solutions of the field equations with cosmological term. Although not identified as such, one of these solutions is, in fact, the static form of the De Sitter solution. To turn this into a solution that is regular everywhere, Weyl pieced together a complete solution out of the static form of the De Sitter solution and the solution for an incompressible fluid. The resulting solution has a zone of matter around the equator. Einstein pointed out (in two letter to Weyl in April 1918) that such hybrid solutions need not be symmetric around the equator, a symmetry Weyl had emphasized and used as an argument for identifying antipodal points. As Weyl was revising the page proofs in response to Einstein's criticism, he considered what would happen if he let the thickness of the zone of matter around the equator go to zero in his combination of the De Sitter solution and the solution for an incompressible fluid. He found that the resulting surface layer of matter has a finite mass. This result, published the following year (*Weyl 1919*) and contested by several later authors, initially seemed to vindicate Einstein's hunch that the De Sitter solution describes a universe very much like his own cylindrical universe, the only difference being that all mass is concentrated on the equator. In his last letter to De Sitter (of April 15, 1918) just before this exchange with Weyl began, Einstein had already announced that Weyl had found a proof for this conjecture. The correspondence with Weyl contains only allusions to this implication of Weyl's work. On May 19, 1918, Weyl wrote to Einstein that the result of the calculation mentioned above "might meet with your approval." Einstein wrote back on May 31 that he was happy that Weyl had finally resolved the "zone issue," and added: "Now the result of your calculation is just what one had to expect."

Einstein's satisfaction was short-lived. On the same day that he wrote these lines to Weyl, Felix Klein sent him a letter, which states, among other things, that the singularity at the equator in the static form of the De Sitter solution is an artifact of the way in which the time coordinate is introduced (see also *Klein 1918*, *1919*). The main point of the letter is something different: Klein wanted to retract his earlier objection (in a letter of April 25, 1918) that Einstein's cosmological model is not time-orientable if antipodal points are identified. Klein had come to realize that he had been conflating the cosmological models of Einstein and De Sitter, and that his objection applied to De Sitter's model rather than Einstein's. Partly because the result concerning the singularity in the De Sitter solution was not emphasized in the letter and partly perhaps because the letter relied heavily on notions from projective geometry (a subject Einstein had done poorly in as a student), Einstein failed to appreciate that Klein's analysis of the De Sitter solution showed that the singularity at the equator can be transformed away and does not indicate the presence of matter after all. In his response, Einstein simply reiterated the argument of his critical note on the De Sitter solution, for which Weyl, he thought, had just provided new support.

In his next letter on the topic (of June 16, 1918), Klein was more direct. As in his earlier letter, he wrote the transformation from the pseudo-Cartesian coordinates of the De Sitter hyper-hyperboloid in 4+1-dimensional Minkowski space-time to the coordinates used to write the solution in static form. This shows, Klein explicitly pointed out, that the singularity at the equator has to be an artifact of the static coordinates. The point can be made more generally than Klein did. Since the De Sitter solution can be represented geometrically as a fully regular hypersurface in a higher-dimensional embedding space, *any* singularity in a coordinate representation of the solution must be an artifact of the coordinates.



Given Klein's transformation from the pseudo-Cartesian coordinates of the embedding space to static coordinates, the properties of such static coordinate systems become fully perspicuous (see the figure to the left: the surface of the hyperboloid represents a 1+1 dimensional [one spatial dimension and one time dimension] De Sitter space-time embedded in a 2+1 dimensional Minkowski space-time). Static coordinates, it turns out, only cover a double-wedge-shaped region of the hyper-hyperboloid (the darkly shaded region), and the hypersurfaces of simultaneity (the ellipses shown in the figure) all intersect on the edge of this wedge, the region Einstein and Weyl called the equator. The singular behavior of the temporal component of the metric, which vanishes on the equator, reflects the fact that in the immediate vicinity of the equator, points infinitesimally close in proper time will be infinitely far removed from one another in coordinate time.[8] The double-wedge-shaped region covered by static coordinates lies fully outside the light cones of points on the equator. This explains why De Sitter concluded that the equator can never be reached.

This time Klein's point was not lost on Einstein. He immediately wrote back to Klein to tell him that he now accepted that the De Sitter solution is matter-free, fully regular, and homogeneous. This does not mean, however, that Einstein also accepted the De Sitter solution as a possible cosmological model. He still held that any acceptable cosmological model would have to be static. Klein had shown that in the static form of the De Sitter solution, the time coordinate breaks down on the equator. In Weyl's hybrid static solution, on the other hand, which coincides with the De Sitter solution outside a zone of matter around the equator, the time coordinate is well defined everywhere. Only this hybrid solution, therefore, provides an acceptable static cosmological model. Einstein thus had to accept that the De Sitter solution forms a counterexample to Mach's principle as he had formulated it in March 1918 and that his critical note on the De Sitter solution stood in need of correction. His modified field equations did allow fully regular matter-free solutions. He could still hold, however, that they did not allow *globally static* fully regular matter-free solutions. From a letter from Weyl to Klein of February 1919, written after consultation with Einstein, it can be inferred that this is the position to which Einstein retreated in response to Klein's analysis of the De Sitter solution. Although Einstein, in a letter of December 1918 to his friend Paul Ehrenfest, one of De Sitter's colleagues in Leyden, expressed his regret that he had unjustly criticized De Sitter, he never published a correction to his critical note on the De Sitter solution.

IV

In the 1920s, the Russian mathematician Alexander Friedmann (1888–1925) and the Belgian priest Georges Lemaître (1894–1966) (see *Lightman 1991*, pp. 20–21) found solutions of Einstein's original field equations (the ones without the cosmological term) that describe an expand-

8. Recall that the temporal component of the metric is basically the number by which the difference in coordinate time between two points or events has to be multiplied to get the difference in proper (physical, real) time between those two events.

ing rather than a static universe. Einstein's reaction to these developments was not unlike his reaction to the De Sitter solution, a mixture of pointing to alleged mathematical difficulties with these solutions (there were none) and of emphasizing that even if one could somehow overcome these difficulties, the solutions could not describe the actual universe. In the late 1920s, the American astronomer Edwin Hubble (1889–1953) (see *Lightman 1991*, p. 27) found irrefutable evidence that our actual universe is expanding. In 1931, Einstein finally accepted the non-static character of the universe and abandoned the cosmological constant, which he later allegedly called the biggest blunder of his life (an anecdote related by physicist George Gamov). Whether or not this last element of the story is apocryphal or not, the development looks rather ironic in the unkind glare of hindsight. If Einstein had stuck to his original field equations of November 1915, he would have predicted that the only way in which his theory allows a spatially closed geometry is through a model in which space-time is expanding. Hubble's discovery would then have been hailed as another triumph for Einstein. Purely on the strength of his conviction that the structure of space-time would have to be fully determined by matter (Mach's principle), he would have made the astonishing prediction that the universe is expanding. As it happened, he not only tinkered with his gravitational field equations to ensure that they be compatible with a static universe, his belief in Mach's principle actually ended up reinforcing his belief in a static universe, since the latter belief provided the only reason he had left for dismissing the anti-Machian De Sitter solution after the dust had settled in his debate of 1916–18 with De Sitter, Weyl, and Klein.

With the recent discovery that the universe is expanding at an accelerating rate, the cosmological constant is once again making a remarkable comeback and this time it looks like it is here to stay. So, as has happened so many times before, Einstein may have the last laugh after all.

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