Quantum Theory
lecture 5: semiclassics

where classical and quantum mechanics meet

- Bohr-Sommerfeld quantization — KP:11.2.0–11.2.1
- WKB approximation — B:14.4—KP:11.1.0
- resonant tunneling — KP:11.3.0
- Landau levels — B:11.3—KP:14.3

B = Ballantine — KP = Konishi & Paffuti

Bohr-Sommerfeld quantization

before the arrival of the Schrödinger equation, Bohr and Sommerfeld had found a way to quantize periodic motion by demanding that the phase accumulated in one period should be an integer multiple of 2π :

$$\frac{1}{\hbar}\oint p\,dx + \gamma = 2\pi n, \quad n = 0, 1, 2, \ldots$$

• $p = mv + qA$ is the canonical momentum (sum of mechanical momentum mv and electromagnetic momentum qA)

• γ is a phase shift picked up at the two turning points

$\gamma = \pm\pi$ at a hard wall, $\gamma = -\pi/2$ at a smooth turning point

• particle in a box: $2pL/\hbar - 2\pi = 2\pi n \Rightarrow p = \pi n\hbar/L, \quad E = \frac{1}{2m}(\pi n\hbar/L)^2$ *(exact)*
triangular potential well: *approximate* (see exercise 5.1)

• more and more accurate in the large-n limit ("semiclassical")

WKB approximation

Wenzel-Kramers-Brillouin (Schrödinger eq.) + Jeffreys (more general diff.eq.) probability amplitude ψ that a particle from $\vec{r}_i$ reaches $\vec{r}_f$ is the sum of the amplitudes along all *classical* paths connecting $\vec{r}_i$ to $\vec{r}_f$

$$\psi = \sum_{\text{paths}} \frac{1}{\sqrt{v(\vec{r}_f)}} \exp\left(\frac{i}{\hbar}\int_{\vec{r}_i}^{\vec{r}_f} \vec{p}\cdot d\vec{l} + i\gamma\right)$$

- factor $1/\sqrt{v} \Rightarrow$ current density $j = v|\psi|^2$ constant along path
- phase shift γ (a.k.a. Maslov index) is π at a hard wall (infinite potential) where $\psi = 0$, so that incident and reflected waves cancel
- at a smooth turning point, v changes sign
$\Rightarrow e^{i\gamma} = 1/\sqrt{-1} = -i \Rightarrow \gamma = -\pi/2$
- sum over paths would be *exact* if we would include also nonclassical paths (Feynman's path integral)

resonant tunneling

see exercise 5.3

Landau levels

see exercise 5.2